SAMPLE QUESTIONS FOR EXAM II

MATH 111, HISTORY OF MATH, DUCHIN

(Typo corrected.)

Some remarks: on both "sides" of the test, you will be given choices of what problems to answer. It is far better to understand most concepts/readings well and skip a few concepts/readings than to cover everything at a lower level of comprehension.

Also, in your studying, review your lecture notes.

Readings

(1) What does Barry Mazur mean by "architectural conjectures"? Give an example of a long-standing open problem in mathematics and briefly (two sentences) discuss its history.

(2) Give an example of the "unreasonable effectiveness of mathematics."

(3) *Each of the following statements is false. Correct it in a few sentences.*

- Euler did not believe in the concept of rigorous proof.
- Hellenistic mathematics used the symbol * for zero.
- Galois gave the first axiomatic definition of a group, which impressed his mentor Descartes.

Problems

(1) Recall that the Prime Number Theorem says that $\pi(x) \sim x/\ln x$. How many primes does this predict between 100 and 200? Between 1100 and 1200? Which is greater? (calculator OK)

(2) It is a fact, provable using trig identities, that $4y^3 - 3y = c$, when $c = \cos\theta$, has a solution $y = \cos(\theta/3)$. This is used to show when an angle trisection is constructible in the Greek manner. Since constructible numbers can only be achieved by rational operations and square roots, the root y is not a constructible number when the cubic polynomail is irreducible.

The polynomial $4y^3 - 3y - \frac{1}{2}$ is irreducible. This proves that a certain angle trisection is not constructible. Which number is shown not to be constructible, and which angle does this show can't be trisected?

(3) True or false: stereographic projection shows that the sphere S^2 is homeomorphic to the plane $\mathbb{R}^2$.

(4) Recall that an algebraic number is a root of a polynomial in $\mathbb{Q}[x]$ and an algebraic integer is a root of a *monic* polynomial in the same ring.

If α is not an algebraic number, can α^2 be an algebraic integer?

(5) Draw an ideal hexagon in the upper half-plane model of the hyperbolic plane. What is its area? (Recall that an ideal polygon has vertices on the boundary, and the area of a triangle is the angle deficit, or π minus the angle sum.)