

## CULTURE-COMPARE CRIB SHEET

MATH 111, HISTORY OF MATH, DUCHIN

The Mesopotamian, Egyptian, Chinese, Indian, and Medieval Islamic mathematical cultures, in sketch. Principal sources include Katz, *A History of Mathematics* and Boyer, *A History of Mathematics*.

### MESOPOTAMIAN

Broadly, the Mesopotamian (or Babylonian) civilization spans from the –4th millenium to Alexander the great (+4th century); more narrowly, –2000 - –600.

Includes modern-day Iraq, plus parts of Syria, Turkey, and Iran. The main site is the plain between the Tigris and Euphrates rivers—the so-called “fertile crescent” that is frequently cited as the “cradle of civilization”—the first recorded literate society. Sometimes small city-states, sometimes united under dynastic control.

Main artifacts: tablets, made of soft clay written on with a stylus and baked solid in the sun. Very durable.

- YBC 7289 (c.–1700) approximates  $\sqrt{2}$  to three sexagesimal places
- tablet from Susa: can infer  $\pi \approx 3;7,30 = 3.125$
- Plimpton 322 (c.–1800) lists Pythagorean triples

Babylonian arithmetic uses sexagesimal notation or base 60, but uses base 10 within sexagesimal places; it is “floating point,” so there is some ambiguity about magnitude, sometimes clarified by a size-word like “thousand” written by a scribe at the end of an expression to hint at the scale.

### EGYPTIAN

Rough dates for dynastic Egypt are –3150 - –31 (when the Roman Empire absorbed Egypt). Agriculture much older. Ruled by kings and priests with an elaborate court.

Fairly poor records survive, mainly papyri—cellulose-based plant products, layered, hammered, dried, and polished.

- Rhind papyrus, 18ft×13in
- Moscow papyrus, 15ft×3in

both are from the –1850 - –1650 range.

Egyptians used a base 10 system, but numbers greater and less than one were *not* treated symmetrically. Numbers less than one were written as sums of distinct unit fractions, and in fact the notation did not allow for more general fractions. Two scripts were used by scribes: *hieroglyphic*, used on walls and stones, with different symbols used for base-10 increments in tally style (like  $\square\square\square$  for 30); and *hieratic*, used in handwriting, with different symbols for the digits 1 – 9.

The multiplication (and division) algorithms were based on doubling and adding; these conjecturally came to Egypt from southwards in Africa. Calculations needed separate tables to resolve  $2/n$  into Egyptian fraction form. Hallmarks of math content were linear equations and proportional reasoning.

Alexandria was a key center of learning, and would become the hub of “Greek” mathematics.

These cultures are both known to have influenced Greek mathematics directly.

## CHINESE

Chinese culture has a continuous artifactual history –1600-now, the longest-running civilization. The first inscriptions were found on banks of Huang (Yellow) River (but stories date back much further). Oldest artifacts are bone inscribed with writing.

China has a long dynastic history, including the following periods which are especially relevant.

- Zhou (–1000 - –500)

The –6th century was a great period for scholarship; though numerous feudal states were at war, academies of scholars were founded in several states and competing lords hired scholar-advisors. This was the age of Confucius, and of technological growth (iron).

- Han (–200 - +200), which constitutes the current ethnic majority

The Han dynasty followed a period of centralization by Emperor Qin, culminating in –221. This era saw the birth of civil service and a standardized education. Several important texts survive from this period. The most important is the *Nine Chapters on the Mathematical Art* (from about the year 0), which codifies mathematical knowledge which was potentially circulating much earlier. Emphasis on perfecting and recapitulating what is known makes origination dates hard to pinpoint.

Chinese number theory was particularly sophisticated, focusing on solving systems of equations. The Chinese Remainder Theorem (treated by Sun Zi in the 3rd century) is particularly important. Later, a Chinese version of Pascal’s triangle was recorded as early as the thirteenth century.

## INDIAN

Earliest records are from the *Harappan* civilization, artifacts of whose writing can be found in the –3rd millenium on the banks of the Indus river. The earliest math records are from the banks of the Ganges river, circa –1000.

Subsequently, a key era for the history of math is the Vedic era (–800 - –600), a religious Hindu period with a stratified social system headed by the Brahmin caste. There was a largely oral tradition through *vedas* (Hindu scriptural texts).

A crucial early document is the *Shatapatha Brahmana* of –800 which contained many temple construction discussions and from which mathematical content can be inferred.

In the –4th century, Alexander the Great’s northern conquests spread Greek influence into India.

Early fifth century documents show extensive work in trigonometry, including the introduction of the sine function in its modern form.

In the +500-600 range, we see documented use of the *kuttaka* or “pulverizer” (=Euclidean algorithm) for treating Pell’s equation and other applications.

Also in this period, we see a gradual and jerky move towards the modern *Hindu-Arabic place value system*. By the ninth century, zero has entered the system as a digit.

In a text from +628, Brahmagupta laid out number theoretic ideas that were extremely influential on Islamic and European mathematics.

In the fifteenth century, Indian scholars worked out the formula  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ , a precursor to the development of series in calculus.

## MEDIEVAL ISLAMIC

Muhammad lived 570–632, and a hugely successful movement followed his death and swept across parts of Asia and Europe.

Return to Baghdad: by circa +750, Baghdad is again a major center of scholarship.

The “House of Wisdom” was established—a scholarly institution with permanent faculty, including al-Khwarizmi (c.780-850). His work was characterized by a move towards *rhetorical algebra*, where systematic, exhaustive algorithmic approaches are detailed for classes of problems. Everything is written out in many cases (to avoid negative numbers)—even the names of numbers are written out. Complicated relationship of algebra and geometry, often with an algebraic solution and a geometric justification.

Later key figure: Omar Khayyam (1050-1123), known for his work in trigonometry and solution of certain cubics; also sophisticated (but inconclusive) investigations on Euclid’s parallel postulate, anticipating non-Euclidean geometry.

Islamic mathematicians extended the Indian numeration with a fully articulated decimal place system. (Hence “Hindu-Arabic place value system.”)