Egyptian arithmetic.

(A1) Use Egyptian algorithms to do the following arithmetic problems: 14×17, 176/16, 37/12.

(A2) Write $\frac{2}{103}$ as the sum of two unequal unit fractions. Show all work. Show that whenever n is a multiple of three, the fraction $\frac{2}{n}$ can be written as a sum of two unequal unit fractions, one of which is $\frac{1}{2n}$.

(B1) Verify that if $(c/a)^2$ is $1 : 33, 45$ and $b = 45$ and $c = 1, 15$, then a, b, c form a Pythagorean triple.

Euclidean algorithm and continued fractions.

(A3) By hand (no calculator), find the continued fraction expansion of 99/21. Using a calculator, find five terms of the continued fraction expansion for $\sqrt[3]{9}$. In both cases, show all steps. Write out the anthyphairesis diagram for 99/21 and explain the correspondence between the diagram and the calculation.

(B2) What is the relationship between the continued fraction expansion of a rational number and that of its reciprocal?

(B3) Suppose a real number α has a completely periodic continued fraction expansion $[a, a, a, a, \ldots]$. Turn this into a quadratic equation in a and α and use this to generate a formula for the values of α. Fine the values of α when $a = 1, 2, 3, 4$.

(B4) Find the continued fraction expansion for $\sqrt{n^2 + 1}$, where n is a natural number. Your answer should be in terms of n. Do the same for $\sqrt{n^2 + n}$.