Some Chinese problems.

(A1) Solve problem 8 of chapter 9 of the *Nine Chapters*: The height of a wall is 10 *ch'ih*. A pole of unknown length leans against the wall so that its top is even with the top of the wall. If the bottom of the pole is moved 1 *ch'ih* further from the wall, the pole will fall to the ground. What is the length of the pole?

(A2) Solve problem 26 of chapter 6 of the *Nine Chapters*: There is a reservoir with five channels bringing in water. If only the first channel is open, the reservoir can be filled in $\frac{1}{3}$ of a day. The second channel by itself will fill the reservoir in one day, the third channel in $2\frac{1}{2}$ days, the fourth one in 3 days, and the fifth one in 5 days. If all the channels are open together, how long will it take to fill the reservoir?

Note: this is the earliest known problem of this type!

the Chinese Remainder Theorem.

(A3) Find a solution to the following Sun Zi-style problem:
If we count by fours, there is a remainder 1.
If we count by fives, there is a remainder 3.
If we count by nines, there is a remainder 6.

You must explain your method; it is not enough to have a numerical solution.

(A4) Give an example of a problem in this style where the p_i are distinct but NOT relatively prime which has no solution. This shows that the "relatively prime" hypothesis in the theorem is necessary.

Calendar-making.

(A5) In the calendar reading, the length of a year is measured to be $365\frac{10463}{43200}$ days. Suppose instead that it had been $365\frac{23}{66}$ days. Explain why the simplest leap year system obtained by continued fractions would have every other year be a leap year, and the next more accurate one would have one leap year out of every three. The next one would call for how many leap years out of every twenty?

YBC 7289.

(A6) A picture of YBC 7289 is included on this handout. If you study the symbols on it, it gives the approximation $1 : 24, 51, 10$ for $\sqrt{2}$. Work out the value of the Babylonian approximation both as a fraction and (using a calculator) as a decimal. Now consider the continued fraction expansion $\sqrt{2} = [1, 2, 2, 2, 2, \ldots]$. Find the first several convergents and see how far you have to go to get an approximation that's better than the Babylonian one. Which one is a more efficient approxmation (that is, has a smaller denominator), the convergent or the Babylonian fraction?

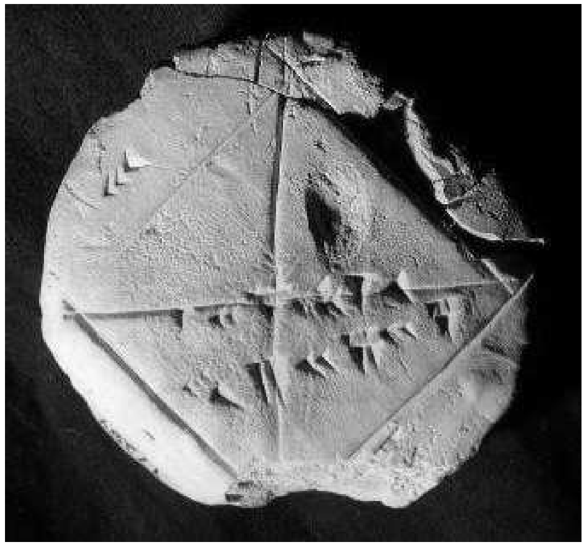

FIGURE 1. YBC 7289

Some numerical algorithms.

(A7) *Think of a number. Now double it. Add nine. Add your original number. Divide by three. Add four. Now subtract your original number. What do you get?*

Try this for a few numbers and write down the answers. Do you always get the same answer in the end? Why?

(A8) Here is an algorithm to decide whether a number is divisible by 9: *take all the digits and add them up to get a new number. If the new number has more than one digit, add them up to get a new number. Keep going until you get a single digit. Your original number was divisible by nine if and only if you end in nine.*

Why does this algorithm work– that is, why does it always terminate after a finite number of steps, and why does it give the right answer?

TIPS FOR THE BOOK PROBLEMS

5.4.1 In other words: the Greeks had a formula for getting a new x and y from an old one. Recognizing this as Brahmagupta composition means filling in the blanks making this true:

$$(x_n, y_n, (-1)^n) * (\quad,\quad,\quad) = (x_{n+1}, y_{n+1}, \quad).$$

5.4.3 The proof that square root of two is irrational is classical: see page 11. It's a little trickier for the square root of N, but similar. The difficulty is that some of the prime factors can appear with powers higher than one. This fact is discussed in the Mazur reading.