Rhetorical algebra.

(A1) al-Khwarizmi writes: *I have divided ten into two parts; I have multiplied the one by ten and the other by itself, and the products were the same.* The first part of the sentence means that he is dealing with two numbers that add up to ten. Solve this problem, which indicates that al-Khwarizmi *was* comfortable dealing with irrational numbers.

Modern techniques would produce two solutions to this problem, but al-Khwarizmi would only have recognized one. Which one and why?

(B1) Write an algebraic equation, write it out in "rhetorical algebra" form, and interpret it as a number trick, like (A7) from the last assignment. Make it a good trick.

Infinity, take one.

(A2) How much is $1 - 1 + 1 - 1 + 1 \cdots$?

On one hand, we get $(1 - 1) + (1 - 1) + (1 - 1) + \cdots = 0$. On the other hand, we get $1 + (-1 + 1) + (-1 + 1) + \cdots = 1$.

So $0 = 1$. Explain what's wrong with this reasoning and compare this to the various proofs that $.9999\ldots = 1$.

(B2) Suppose that you're walking on a road and every mile you go you come to another fork in the road. Suppose the roads go on forever. Consider the set of all paths (that is, all choices of L or R at every fork; so one path is LLLLLL...). Is this set in bijection with $\mathbb{N}$ (in other words *countable*) or is it in bijection with the interval $[0, 1]$ (and therefore *uncountable*)?

TIPS FOR THE BOOK PROBLEMS

4.3.1 The figure is meant to suggest that the triangles have been approximated with rectangles, in each case, of height $h/4$. There's more than one way to pass to the "next step" in this exhaustion process, but a natural one is to divide up the triangles using rectangles of height h/n. Then you need to show, using the fact that the area is $bh/2$, that these rectangular approximations get arbitrarily close to the true area.

4.4.1-3 Do NOT use properties of the logarithm function to answer these questions—you are trying to derive properties of the logarithm function from this unusual definition, so you should only use facts about area.