Paradoxes, reference, self-reference.

(A1) Let's define some new words: *homodescriptual* and *heterodescriptual*. A word is called homodescriptual if it describes itself, like "word," or "ostentatious," and it's called heterodescriptual if it doesn't. First list several examples of homodescriptual words and examples of heterodescriptual words (be creative!). Then find a Russell-type paradox using these concepts, and explain the connection.

(B1) Consider the following sentence.

If this sentence is true, then I owe $5 to Moon Duchin.

If the sentence is true, what does that tell you about our financial relationship? Then doesn't that make the sentence true? Discuss. (P.S. I take cash or money order.)

Formulas and symbols.

(A2) In Greek mathematics, some scholars (like Ptolemy, working in the +2nd century in Alexandria) studied trigonometric functions, particularly in connection with problems in astronomy. They did not use the modern sine and cosine, though, but rather mainly a function called the *chord*.

The chord of an angle α, denoted $crd(\alpha)$, is the distance between two points on a circle of radius R such that the angle between them at the center of the circle is α. (So notice: the value of the chord depends on both α and R.)

Let R be fixed. Work out $crd(\alpha)$ in terms of α, R and the modern sine function. Work out $R\sin(\alpha)$ in terms of $crd(\alpha)$.

(B2) Derive the following infinite series expression for π by using the Taylor series for arctan:

$$\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots).$$

What is the the fourteenth partial sum of this series? How does it compare to the continued fraction convergents?
Extra credit: if you know how to program, figure out how far you'd have to expand the series to get a better approximation than the fourth continued fraction convergent, 355/113.

Tips for the book problems

6.6.1 For all three of these problems, note that equation (3) is true for all n. (He's just saying you can get a particular consequence from it in the case $n = 4m + 1$.)

This one is tricky but very satisfying. Don't look at these hints unless you are stumped for ten minutes! And if you're going to use these hints, explain why they work. (There are other ways to solve the problems as well.)

Hint: let $\theta = \pi/2 - \alpha$. This allows you to rewrite the left-hand side of the formula as $(\operatorname{cis}\alpha)^n$, and then apply (3).

Another Hint: the cases $n = 4m + 1$ and $n = 4m + 3$ determine whether $n\pi/2$ is the same angle as $\pi/2$ or $3\pi/2$. (Do you see why?)

Final Hint: $\sin(\pi + A) = -\cos(A)$ and $\cos(\pi + A) = -\sin(A)$. (Do you see why?)

6.6.2-3 It might help to use the fact that complex conjugates commute with exponentiation:

If $(a + bi)^k = c + di$, then $(a - bi)^k = c - di$.

In particular, this means that the nth root of $(\cos nA - i \sin nA)$ is the conjugate of the nth root of $(\cos nA + i \sin nA)$.