COMMENTS ON EXAM SOLUTIONS

Question 2 Integers are whole numbers. To put a square root $\sqrt{n}$ between two whole numbers, figure out which two perfect squares are on either side of n.

For instance,

$$\sqrt{36} < \sqrt{40} < \sqrt{49} \quad \Longrightarrow 6 < \sqrt{40} < 7.$$

Also for this question, to get $[[5\pi]]$, you should first multiply 5 times π and then round down. $\pi \approx 22/7$, so $5\pi \approx 110/7 = 15\frac{5}{7}$. Therefore $[[5\pi]] = 15$.

Question 3 $|-x| = x$ is FALSE because the left-hand side is always ≥ 0, while the right-hand side can be either positive or negative. Another way to see it is to note that the function $f(x) = |-x|$ does NOT always give output that is the same as its input (for instance $f(-12) = 12$) so it is not the same function as $f(x) = x$.

Question 4 It is not, not, not, not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. This was by far the biggest mistake on this problem.

Simplify $\sqrt{(x^3+1)^2 + 2x^3 + 3}$ as far as possible: just multiplying out the polynomial gives the answer $\sqrt{x^6 + 4x^3 + 4}$. *That answer received full credit.*

But it's also true that $x^6 + 4x^3 + 4 = (x^3+2)^2$. So the answer can be further simplified to

$$\sqrt{x^6 + 4x^3 + 4} = \sqrt{(x^3+2)^2} = |x^3 + 2|.$$

Question 5 Note that the domain of a function must equal the range of its inverse function! This is true because taking the inverse function exchanges the roles of input and output. (*Domain* is all possible inputs to a function and *range* is all outputs that are achieved.)

Question 7 This question was testing whether you could compute limits, and it was also testing whether you understood the relationship between limits and asymptotes. Every kind of asymptote can be exhibited by a certain limit. For instance, a vertical asymptote always corresponds to some one-sided limit equaling $\pm\infty$. A horizontal asymptote always corresponds to a limit as the variable goes to infinity equaling a finite number.

So if you have the answer $\lim_{x\to 1} \frac{x^2+x-2}{x-1} = 3$, that tells you **there is no asymptote** at $x = 1$. (It's a removable discontinuity instead.)

On the other hand, $\lim_{x\to 1} \frac{x^2-x-2}{x-1} = \pm\infty$ (you get $-\infty$ for one of the one-sided limits and ∞ for the other), so that function **does** have a vertical asymptote at $x = 1$.

How about $\lim_{x\to 5^-} \frac{2^x}{1-x}$? Easy—the function is continuous at the point (it's a quotient of continuous functions, and it's defined at $x = 5$) so you simply plug in $x = 5$ to get the answer $32/(-4) = -8$. (**No asymptote there**.)

For the last part of this question, you needed to come up with a function that has a vertical asymptote at $x = 10$. To do this, you should have at least a one-sided limit at $x = 10$ giving $\pm\infty$. The three answer sheets in the solution set give three different possibilities.

Question 8 Here, you want to have the function be undefined at $x = c$ but equal to $g(x)$ everywhere else. Just let

$$f(x) = \frac{g(x) \cdot (x - c)}{x - c}.$$

I gave lots of partial credit on this one (for instance, for showing the graph, defining a removable discontinuity, or even explaining how to make the function have a discontinuity at all).