

SECOND MIDTERM

Question 1 Find these derivatives. No need to simplify.

$$\frac{d}{dx}(10x^5 - 32x^2) = 50x^4 - 64x$$

$$(\sin(2x))''' = -8 \cos(2x)$$

The seventh derivative of $x^5 + e^x$. e^x

$$\frac{d}{dy}(570006 + yx^5 + 1/\pi) = x^5$$

$$(\tan x)' = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

(use quotient rule)

rewrite: $u^{-6/5}$

$$\frac{d}{du} \left(\frac{1}{u^{6/5}} \right) = \frac{-6}{5} u^{-11/5}$$

$$\frac{d}{dx}(x \cdot \sin x) = x(\cos x) + 1(\sin x)$$

$$\frac{d}{dr}(\pi r^2) = 2\pi r$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt[3]{8-x}} \right) = \frac{-1}{3} (8-x)^{-4/3} \cdot (-1)$$

$$\frac{d}{dx}(\sqrt{2x}) = \frac{1}{2} (2x)^{-1/2} \cdot (2)$$

Question 2 Your benefit from studying t hours for your calculus exam is given by the function $B(t)$.

What is your marginal benefit when $t = 12$?

$$B'(12)$$

What does it mean if that value is negative?

That, at the instant that you've been studying for 12 hours, your benefit is actually decreasing with further study.

What is the meaning of $\frac{B(8) - B(6)}{2}$?

The average rate of change of benefit on the interval $[6, 8]$.

Question 3 Let $f(x) = (1 - x)^2$. We'll use h as a variable to represent a small number.

What is $f(4)$?

$$9$$

What is $f(4 + h)$?

$$9 + 6h + h^2$$

Let $A = \frac{f(4+h) - f(4)}{h}$. Find A and simplify your answer.

$$A = 6 + h$$

Switching gears, use any of the rules of differentiation to find the derivative. Simplify.

$$f'(x) = -2 + 2x$$

Let $B = f'(4)$. What is B ?

$$B = 6$$

True or False: $B = \lim_{h \rightarrow 0} A$.

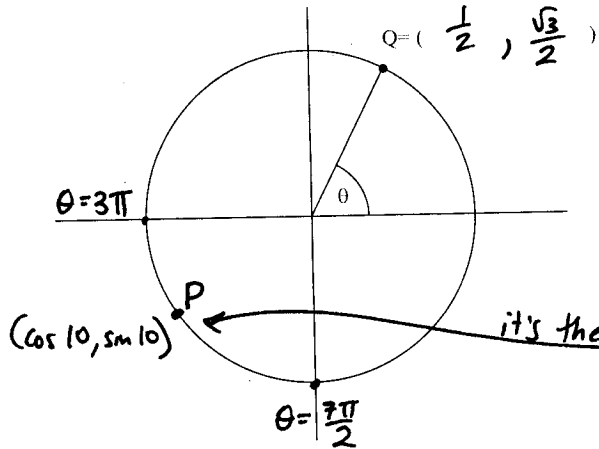
T

Explain.

$$\lim_{h \rightarrow 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_A = \underbrace{f'(x)}_B \quad \text{Definition of derivative!}$$

Question 4

For this problem, remember that $\pi \approx 3.14$ (or if you prefer, $\pi \approx \frac{22}{7}$).



- For the angle $\theta = \frac{\pi}{3}$, what are the coordinates of the corresponding point Q ? (Fill in on the diagram.)
- Label the point P on the unit circle that you would use to help you figure out $\sin(10)$ and $\cos(10)$.
- Is $\sin(10)$ positive or negative?

negative

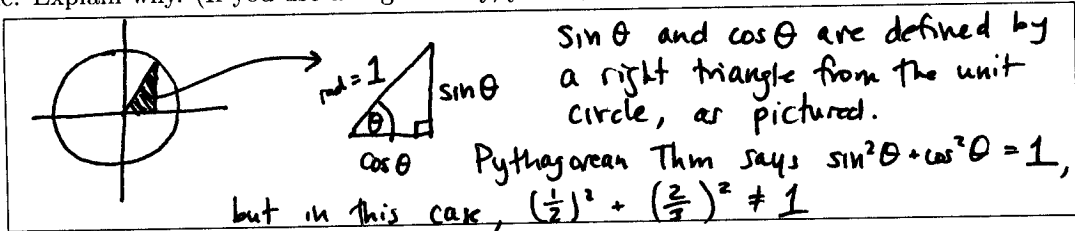
Let the horizontal position of a point at time t be given by $h(t) = \cos t$, while the vertical position is given by $v(t) = \sin t$. (Like on an Etch-a-sketch.)

Are $h'(10)$ and $h''(10)$ positive or negative? What does that tell you about the movement of the point?

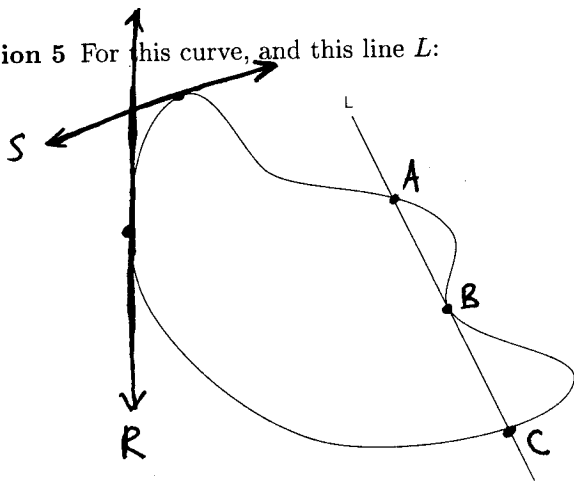
$$\begin{aligned} h'(t) &= -\sin t \Rightarrow h'(10) = -\sin 10 \text{ POSITIVE} && \text{since } h \text{ is} \\ h''(t) &= -\cos t \Rightarrow h''(10) = -\cos 10 \text{ POSITIVE} && \text{horizontal movement,} \end{aligned}$$

at time $t=10$, the point is moving **RIGHT** and speeding up!
→ (accelerating)

Your neighbor tells you he knows an angle θ such that $\sin(\theta) = 1/2$ and $\cos(\theta) = 2/3$. You know this is impossible. Explain why. (If you use a trig identity, you must explain why it's true.)



Question 5 For this curve, and this line L :



Is L a secant line, a tangent line, or both? Explain.

Both! Secant because it crosses the curve more than once, tangent because it approximates the curve at B .

- Draw a tangent line R that is vertical.
- Draw a tangent line S that has slope approximately $1/2$.

SECOND MIDTERM

Question 1 Find these derivatives. No need to simplify.

$$\frac{d}{dx}(20x^4 - 8x^{10}) = 80x^3 - 80x^9$$

$$(\sin(3x))''' = -27 \cos(3x)$$

The seventh derivative of $x^5 + e^x$. e^x

$$\frac{d}{dy}(7002y + xy^2 + 1/\pi) = 7002 + 2xy$$

$$(\tan x)' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

(use quotient rule)

rewrite: $2u^{-3/4}$

$$\frac{d}{du} \left(\frac{2}{u^{3/4}} \right) = -\frac{3}{2} u^{-7/4}$$

$$\frac{d}{dx}(x^2 \cdot \cos x) = x^2(-\sin x) + (2x)\cos x$$

$$\frac{d}{dr}(\pi r^2) = 2\pi r$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt[3]{5+2x}} \right) = -\frac{1}{3}(5+2x)^{-4/3} \cdot (2)$$

$$\frac{d}{dx}(\sqrt{2x}) = \frac{1}{2}(2x)^{-1/2} \cdot (2)$$

Question 2 Your benefit from studying t hours for your calculus exam is given by the function $B(t)$.

See previous

What is your marginal benefit when $t = 12$?

What does it mean if that value is negative?

What is the meaning of $\frac{B(8) - B(6)}{2}$?

Question 3 Let $f(x) = (2 - x)^2$. We'll use h as a variable to represent a small number.

What is $f(4)$? 4

What is $f(4 + h)$? $4 + 4h + h^2$

Let $A = \frac{f(4+h) - f(4)}{h}$. Find A and simplify your answer.

$A = 4 + h$

Switching gears, use any of the rules of differentiation to find the derivative. Simplify.

$$f'(x) = -4 + 2x$$

Let $B = f'(4)$. What is B ?

$B = 4$

True or False: $B = \lim_{h \rightarrow 0} A$.

T

Explain.

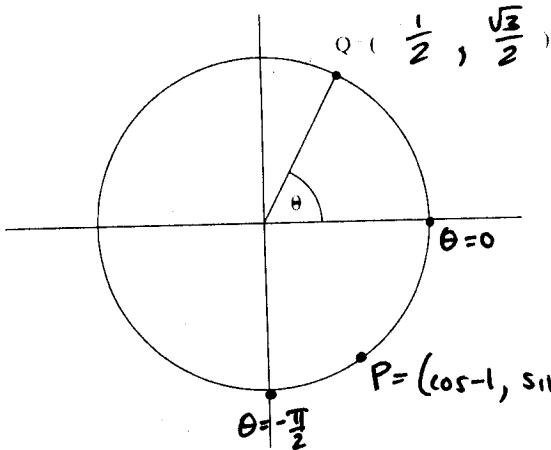
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

Definition of derivative!

$$\lim_{h \rightarrow 0} A = B$$

Question 4

For this problem, remember that $\pi \approx 3.14$ (or if you prefer, $\pi \approx \frac{22}{7}$).



- For the angle $\theta = \frac{\pi}{3}$, what are the coordinates of the corresponding point Q ? (Fill in on the diagram.)
- Label the point P on the unit circle that you would use to help you figure out $\sin(-1)$ and $\cos(-1)$.
- Is $\sin(-1)$ positive or negative?

negative

Let the horizontal position of a point at time t be given by $h(t) = \cos t$, while the vertical position is given by $v(t) = \sin t$. (Like on an Etch-a-sketch.)

Are $h'(-1)$ and $h''(-1)$ positive or negative? What does that tell you about the movement of the point?

$$h'(t) = -\sin t \Rightarrow h'(-1) = -\sin(-1) \text{ POSITIVE} \quad h \text{ is horizontal movement}$$

$$h''(t) = -\cos t \Rightarrow h''(-1) = -\cos(-1) \text{ NEGATIVE}$$

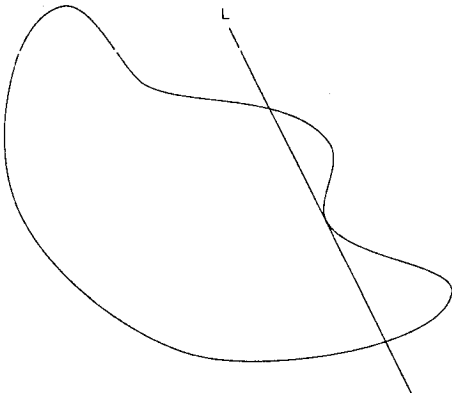
at time $t = -1$, point is moving RIGHT and slowing down!

Your neighbor tells you he knows an angle θ such that $\sin(\theta) = 1/2$ and $\cos(\theta) = 2/3$. You know this is impossible. Explain why. (If you use a trig identity, you must explain why it's true.)

See previous

Question 5 For this curve, and this line L :

See previous



Is L a secant line, a tangent line, or both? Explain.

- Draw a tangent line R that is vertical.
- Draw a tangent line S that has slope approximately $1/2$.

Question 1 Find these derivatives. No need to simplify.

$$\frac{d}{dx}(6x^{12} + 7x^9) = 72x^{11} + 63x^8$$

rewrite: $3u^{-4/3}$

$$\frac{d}{du}\left(\frac{3}{u^{4/3}}\right) = -4u^{-7/3}$$

$$(\sin(3x))''' = -27 \cos(3x)$$

$$\frac{d}{dx}(x \cdot \cos x) = x(-\sin x) + 1(\cos x)$$

The seventh derivative of $x^5 + e^x$.

$$e^x$$

$$\frac{d}{dr}(\pi r^2) = 2\pi r$$

$$\frac{d}{dy}(5899 + x^2y + 1/\pi) = x^2$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt[3]{6-3x}}\right) = -\frac{1}{3}(6-3x)^{-4/3} \cdot (-3)$$

$$(\tan x)' = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} = \sec^2 x$$

(use quotient rule)

$$\frac{d}{dx}(\sqrt{2x}) = \frac{1}{2}(2x)^{-1/2} \cdot (2)$$

rewrite: $\frac{\sin x}{\cos x}$

rewrite: $(2x)^{1/2}$

Question 2 Your benefit from studying t hours for your calculus exam is given by the function $B(t)$.

What is your marginal benefit when $t = 12$?

$$B'(12)$$

What does it mean if that value is negative?

Benefit is decreasing with small further study at time $t=12$. (Instantaneous rate of change is negative.)

What is the meaning of $\frac{B(8) - B(6)}{2}$?

Average rate of change of benefit over the time interval $[6, 8]$.
OR slope of the secant line btw $(6, B(6))$ and $(8, B(8))$

Question 3 Let $f(x) = (3 - x)^2$. We'll use h as a variable to represent a small number.

What is $f(4)$?

$$1$$

What is $f(4 + h)$?

$$1 + 2h + h^2$$

Let $A = \frac{f(4+h) - f(4)}{h}$. Find A and simplify your answer.

$$A = 2 + h$$

Switching gears, use any of the rules of differentiation to find the derivative. Simplify.

$$f'(x) = 2x - 6$$

Let $B = f'(4)$. What is B ?

$$B = 2$$

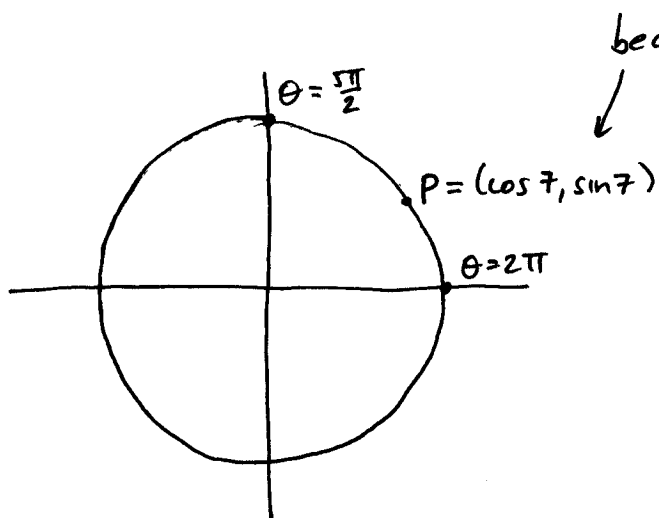
True or False: $B = \lim_{h \rightarrow 0} A$.

T

Explain.

A is the slope of the secant line between $(4, 1)$ and a nearby point. B is the slope of the tangent at $(4, 1)$. $\lim_{h \rightarrow 0} A = B$ by def. of derivative!

Question 4



because $2\pi < 7 < \frac{5\pi}{2}$

so $\cos 7 > 0$ (POS)

and $\sin 7 > 0$ (POS)

$$h'(t) = -\sin t \Rightarrow h'(7) = -\sin(7) \text{ NEGATIVE}$$

$$h''(t) = -\cos t \Rightarrow h''(7) = -\cos(7) \text{ NEGATIVE}$$

h is horizontal movement, so this means
at time $t=7$, the point is moving LEFT and, because its
acceleration is negative, speeding up leftwards.

[rest of Question 4 plus Question 5: see previous versions.]