

MATH 180, PROBLEM SET 1

- (1) For real numbers  $a, b, c, d \geq 0$ , prove that

$$\frac{a}{b} < \frac{c}{d} \implies \frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}.$$

- (2) Find the number whose continued fraction expansion is  $[3, 1, 4, 1, 5]$ . Also find the continued fraction expansion for  $99/21$ . Show your steps.
- (3) Consider the square grid of sidelength one where the horizontal edges are labeled  $b$  and the vertical edges are labeled  $a$ . For lines starting at the origin, write down the *cutting sequence* of  $a$ 's and  $b$ 's as they cross the edges of the grid. If they cross at a vertex exactly, mark down  $ab$ . So the cutting sequence for the line of slope  $3/2$  is

$$(babab)(babab)(babab)\dots$$

For each of the following slopes, either find the full cutting sequence until it repeats, or find enough terms to get an idea of the pattern.

- (a) 4, (b)  $11/5$ , (c)  $9/7$ , and (d)  $\phi = \frac{1+\sqrt{5}}{2}$ .