

## MATH 180, MINITEST 4

Choose two.

- (1) Consider the dynamical system given by  $T(x) = 2x \pmod{1}$  acting on the interval  $[0, 1)$ . If an orbit  $x, Tx, T^2x, \dots$  is coded over the alphabet  $\{0, 1\}$  (according to whether each element is less than or more than  $1/2$ ), give a purely periodic code sequence corresponding to a value of  $x$  strictly between 0.8 and 0.9. For your sequence, what is the smallest value of  $n$  such that  $T^n x = x$ ?
- (2) What is the area of an ideal octagon (all eight vertices on the boundary and geodesic sides) in the hyperbolic plane? What is the area of a regular geodesic octagon in  $\mathbb{H}$  all of whose angles measure  $\pi/4$ ?
- (3) Consider  $A_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 7 \\ 0 & 1 \end{pmatrix}, A_3 = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} \in SL_2(\mathbb{Z})$ . Where does each of them take the Ford circle  $C(0/1)$ ? (Recall that a Ford circle  $C(p/q)$  is tangent to the real axis at  $p/q$  and has diameter  $1/q^2$ .)