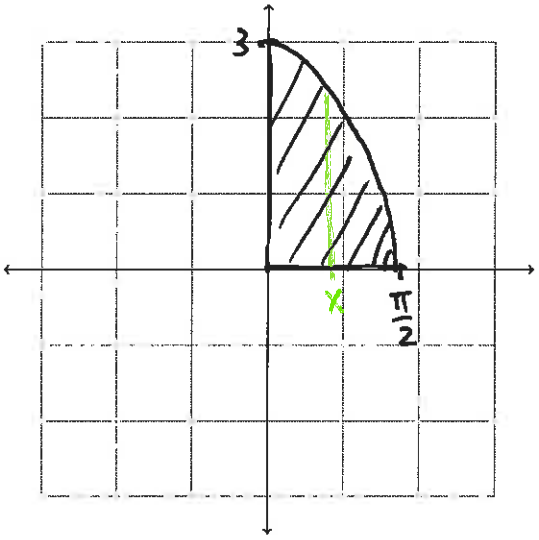
QUIZ 1 Math 34 (Calc II), Spring 2013

Consider the region bounded by $y = 3\cos x$ and the x and y axes, for $0 \le x \le \pi/2$. When revolved around the x-axis, it makes a solid.

Sketch the region.



$r = 3\cos x$

$A = \pi r^2 = 9\cos^2 x \cdot \pi$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

$\sin \pi = 0$

$\sin 0 = 0$

With the method of [disks] and integration with respect to [x].

disks / washers / shells *x/y*

compute the volume of the solid below, showing your work.

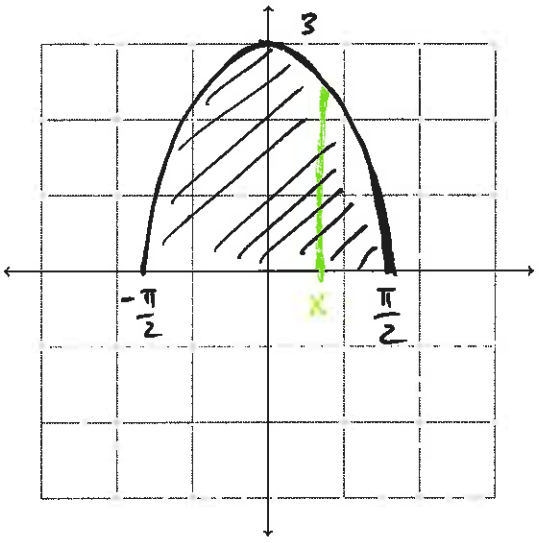
$$V = \int_0^{\pi/2} 9\pi \cos^2 x \, dx = 9\pi \int_0^{\pi/2} \frac{1+\cos(2x)}{2} \, dx$$

$$= \frac{9\pi}{2}\left[x + \frac{\sin(2x)}{2}\right]_0^{\pi/2} = \frac{9\pi}{2}\left[\frac{\pi}{2} - 0\right] = \frac{9\pi^2}{4}.$$

QUIZ 1 Math 34 (Calc II), Spring 2013

Consider the region bounded by $y = 3\cos x$ and the x-axis, for $-\pi/2 \le x \le \pi/2$. When revolved around the x-axis, it makes a solid.

Sketch the region.



$r = 3\cos x$

$A = \pi r^2 = 9\cos^2 x \cdot \pi$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

With the method of [disks] (*disks / washers / shells*) and integration with respect to [x] (x/y),

compute the volume of the solid below, showing your work.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 9\pi\cos^2 x\,dx = \frac{9\pi}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\cos(2x))\,dx$$

$$= \frac{9\pi}{2}\left[x + \frac{\sin(2x)}{2}\right]_{-\frac{\pi}{2}}^{\pi/2} = \frac{9\pi}{2}\left[\frac{\pi}{2} - \frac{-\pi}{2}\right] = \frac{9\pi^2}{2}.$$

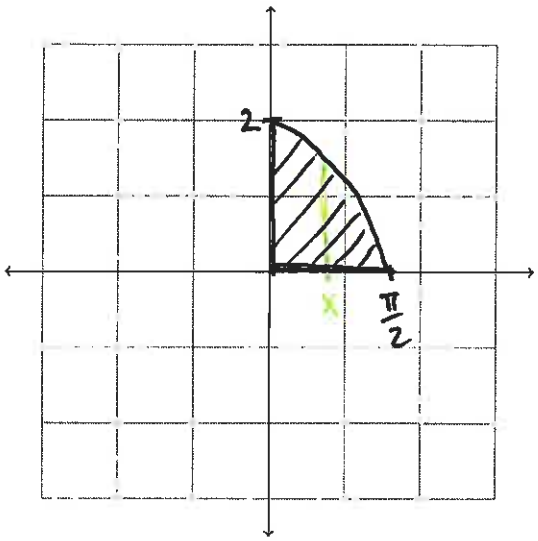
$\sin\pi = 0$

$\sin(-\pi) = 0$

QUIZ 1 Math 34 (Calc II), Spring 2013

Consider the region bounded by $y = 2\cos x$ and the x and y axes, for $0 \le x \le \pi/2$. When revolved around the x-axis, it makes a solid.

Sketch the region.



$r = 2\cos x$

$A = \pi r^2 = 4\cos^2 x \cdot \pi$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

$\sin \pi = 0$

$\sin(0) = 0$

With the method of [disks] and integration with respect to [x]

disks / washers / shells *x/y*

compute the volume of the solid below, showing your work.

$$V = \int_0^{\pi/2} 4\pi\cos^2 x\,dx = 4\pi\int_0^{\pi/2} \frac{1+\cos(2x)}{2}\,dx$$

$$= 2\pi\left[x + \frac{\sin(2x)}{2}\right]_0^{\pi/2} = 2\pi\left[\frac{\pi}{2} - 0\right] = \pi^2.$$