NAME, SECTION

Evaluate the following integral or state that it diverges.

$$\int_0^\infty \frac{\arctan(2x)}{1+4x^2}\,dx$$

**Step 1:** Find the antiderivative.

$u = \tan^{-1}(2x)$

$du = \dfrac{2dx}{1+4x^2}$

$$\int \frac{\arctan(2x)}{1+4x^2}\,dx = \int \frac{1}{2} u\,du = \frac{u^2}{4} + C$$

$$= \frac{(\arctan(2x))^2}{4} + C$$

**Step 2:** Using that answer, set up the improper integral as a limit.

$$\int_0^\infty \frac{\arctan(2x)}{1+4x^2}\,dx = \lim_{b\to\infty} \int_0^b \frac{\arctan(2x)}{1+4x^2}\,dx$$

**Step 3:** Evaluate the limit, showing your work. You must evaluate, but you don't need to simplify.

$$\lim_{b\to\infty} \int_0^b \frac{\arctan(2x)}{1+4x^2}\,dx = \lim_{b\to\infty} \frac{(\arctan(2b))^2}{4} + \frac{(\arctan(0))^2}{4}$$

$$= \frac{\left(\frac{\pi}{2}\right)^2}{4} + 0$$

$$= \frac{\pi^2}{16}$$

Name, Section

Evaluate the following integral or state that it diverges.

$$\int_{-1}^{\infty} \frac{\arctan x}{x^2+1}\,dx$$

**Step 1:** Find the antiderivative.

$$\int \frac{\arctan x}{x^2+1}\,dx =$$

$u = \arctan x$

$du = \frac{1}{1+x^2}\,dx$

$$\int \frac{\arctan x}{x^2+1}\,dx = \int u\,du$$
$$= u^2/2 + C$$
$$= (\arctan x)^2/2 + C$$

**Step 2:** Using that answer, set up the improper integral as a limit.

$$\int_{-1}^{\infty} \frac{\arctan x}{x^2+1}\,dx = \lim_{b\to\infty} \int_{-1}^{b} \frac{\arctan x}{x^2+1}\,dx$$

**Step 3:** Evaluate the limit, showing your work. You must evaluate, but you don't need to simplify.

$$\lim_{b\to\infty} \int_{-1}^{b} (\arctan x)/(x^2+1)\,dx = \lim_{b\to\infty} \frac{(\arctan(x))^2}{2} - \frac{(\arctan(-1))^2}{2}$$
$$= \left(\frac{\pi}{2}\right)^2 - \left(-\frac{\pi}{4}\right)^2$$

NAME, SECTION

Evaluate the following integral or state that it diverges.

$$\int_1^\infty \frac{2\arctan x}{x^2+1}\,dx$$

**Step 1:** Find the antiderivative.

$$\int \frac{2\arctan x}{x^2+1}\,dx =$$

$u = \arctan x$

$du = \frac{1}{x^2+1}\,dx$

$$\int \frac{2\arctan x}{x^2+1}\,dx = \int 2u\,du$$

$$= u^2 + C$$

$$= (\arctan(x))^2 + C$$

**Step 2:** Using that answer, set up the improper integral as a limit.

$$\int_1^\infty \frac{2\arctan x}{x^2+1}\,dx = \lim_{b\to\infty} \int_1^b \frac{2\arctan x}{x^2+1}\,dx$$

**Step 3:** Evaluate the limit, showing your work. You must evaluate, but you don't need to simplify.

$$\lim_{b\to\infty} \int_1^b \frac{2\arctan x}{x^2+1}\,dx = \lim_{b\to\infty} \arctan(b)^2 - \arctan(1)^2$$

$$= \left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{4}\right)^2$$