Quiz 5 Math 34 (Calc II), Spring 2013

Name, Section

(1) Do the following sequences converge or diverge? Justify briefly.

$$\{\sin(.98^n)\}_{n\geq 1}$$

Since $|.98|<1$ $\lim_{n\to\infty} .98^n = 0.$

Then, since sin is continuous

$$\lim_{n\to\infty} \sin(.98^n) = \sin\left(\lim_{n\to\infty} .98^n\right) = \sin(0) = 0$$

So the sequence converges.

$$\left\{\frac{\cos^2(4n)}{\sqrt{\ln n}}\right\}_{n\geq 10}$$

Notice $0 \leq \cos^2(4n) \leq 1$. Then

$$0 \leq \frac{\cos^2(4n)}{\sqrt{\ln n}} \leq \frac{1}{\sqrt{\ln n}}.$$

Since $\lim_{n\to\infty} \frac{1}{\sqrt{\ln n}} = 0$, the Squeeze theorem says

$\lim_{n\to\infty} \frac{\cos^2(4n)}{\sqrt{\ln n}} = 0$ so the sequence converges.

(2) Does the following infinite series converge or diverge?
If it converges, find its sum.

$$\sum_{k=0}^{\infty} 21\left(\frac{2}{\pi}\right)^k$$

This is a geometric series with $|\frac{2}{\pi}|<1$ so it converges and $\sum_{k=0}^{\infty} 21\left(\frac{2}{\pi}\right)^k = \frac{21}{1-\frac{2}{\pi}}$

QUIZ 5 Math 34 (Calc II), Spring 2013

NAME, SECTION

(1) Do the following sequences converge or diverge? Justify briefly.

$$\{\cos(.99^n)\}_{n\geq 1}$$

$\lim_{n\to\infty} .99^n = 0$ since $|.99| < 1$

Since Cos is continuous:

$\lim_{n\to\infty} \cos(.99^n) = \cos\left(\lim_{n\to\infty} .99^n\right) = \cos(0) = 1$

So the sequence converges.

$$\left\{\frac{\sin^2(3n)}{\sqrt{\ln n}}\right\}_{n\geq 10}$$

Notice $0 \leq \sin^2(3n) \leq 1$. Then

$0 \leq \frac{\sin^2(3n)}{\sqrt{\ln n}} \leq \frac{1}{\sqrt{\ln n}}$.

Since $\lim_{n\to\infty} \frac{1}{\sqrt{\ln n}} = 0$, the squeeze theorem says

$\lim_{n\to\infty} \frac{\sin^2(3n)}{\sqrt{\ln n}} = 0$ So the sequence converges.

(2) Does the following infinite series converge or diverge?
If it converges, find its sum.

$$\sum_{k=0}^{\infty} 2\left(\frac{\pi}{6}\right)^k$$

This is a geometric series with $\left|\frac{\pi}{6}\right| < 1$ so

it converges and $\sum_{k=0}^{\infty} 2\left(\frac{\pi}{6}\right)^k = \frac{2}{1-\frac{\pi}{6}}$.

QUIZ 5 Math 34 (Calc II), Spring 2013

NAME, SECTION

(1) Do the following sequences converge or diverge? Justify briefly.

$$\{\cos(.97^n)\}_{n\geq 1}$$

Since $|.97| < 1$ $\lim_{n\to\infty} .97^n = 0$.

Then, since cos is continuous

$\lim_{n\to\infty} \cos(.97^n) = \cos(\lim_{n\to\infty} .97^n) = \cos(0) = 1$

so the sequence converges.

$$\left\{\frac{\sin^2(7n)}{\sqrt{\ln n}}\right\}_{n\geq 10}$$

Notice $0 \leq \sin^2(7n) \leq 1$. Then

$0 \leq \frac{\sin^2(7n)}{\sqrt{\ln n}} \leq \frac{1}{\sqrt{\ln n}}$.

Since $\lim_{n\to\infty} \frac{1}{\sqrt{\ln n}} = 0$, the squeeze theorem says that

$\lim_{n\to\infty} \frac{\sin^2(7n)}{\sqrt{\ln n}} = 0$ so the sequence converges.

(2) Does the following infinite series converge or diverge?
If it converges, find its sum.

$$\sum_{k=0}^{\infty} 5\left(\frac{\pi}{12}\right)^k$$

This is a geometric series with $\left|\frac{\pi}{12}\right| < 1$ so it converges and $\sum_{k=0}^{\infty} 5\left(\frac{\pi}{12}\right)^k = \frac{5}{1-\frac{\pi}{12}}$