QUIZ 6 Math 34, Spring 2013

NAME, SECTION

(1) Apply the divergence test to each of these and report the conclusions. Justify briefly.

$\sum_{k=2}^{\infty} \frac{1}{k}$

Since $\lim_{k \to \infty} \frac{1}{k} = 0$, the divergence test is inconclusive.

$\sum_{k=2}^{\infty} \frac{\sqrt{k}}{\ln k}$

We check $\lim_{x \to \infty} \frac{\sqrt{x}}{\log x} \overset{\frac{\infty}{\infty}}{=} \lim_{x \to \infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{x}{2\sqrt{x}} = \lim_{x \to \infty} \frac{\sqrt{x}}{2} = \infty$

Therefore, $\lim_{k \to \infty} \frac{\sqrt{k}}{\ln k} = \infty \neq 0$, so $\sum_{k=2}^{\infty} \frac{\sqrt{k}}{\ln k}$ diverges by the divergence test.

(2) Suppose you want to study the sum $\sum_{k=1}^{\infty} ke^{-2k^2}$ using the integral test.

(a) List the hypotheses to check for the integral test. (You do not need to check them.)
(b) What continuous function $f(x)$ would you use to study the behavior of this series?
(c) Write down the improper integral you would use in the test. (Don't evaluate it.)
(d) Explain how that integral will provide a conclusion about the original series.

(a) We must check that xe^{-2x^2} is continuous, decreasing and positive on $[1, \infty)$

(b) $f(x) = xe^{-2x^2}$

(c) $\int_1^{\infty} xe^{-2x^2} dx$

(d) If the integral converges then the series will converge. If the integral diverges then the series will diverge.