QUIZ 7 Math 34, Spring 2013

NAME, SECTION

(1) Use the (direct/basic/ordinary) comparison test to determine whether $\sum_{n=1}^{\infty} \frac{n+4^n}{3^n}$ converges or diverges.

Notice $\frac{n+4^n}{3^n} = \frac{n}{3^n} + \frac{4^n}{3^n}$ and $\frac{n}{3^n} > 0$ for all $n \in \mathbb{N}$.

Therefore, $\frac{4^n}{3^n} < \frac{n+4^n}{3^n}$ for all $n \in \mathbb{N}$.

Since $\sum_{n=1}^{\infty} \frac{4^n}{3^n}$ is a geometric series with $\frac{4}{3} > 1$, $\sum_{n=1}^{\infty} \frac{4^n}{3^n}$ diverges and by the direct comparison test so does $\sum_{n=1}^{\infty} \frac{n+4^n}{3^n}$.

(2) Use the limit comparison test to determine whether $\sum_{n=1}^{\infty} \frac{n^2+3n}{30n^3+1}$ converges or diverges.

Let $a_n = \frac{n^2+3n}{30n^3+1}$ and let $b_n = \frac{1}{n}$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3+3n^2}{30n^3+1} = \lim_{n\to\infty} \frac{n^3+3n^2}{30n^3+1} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{1+\frac{3}{n}}{30+\frac{1}{n^3}} = \frac{1}{30}.$$

Since $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{30}$ we know that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or diverge together. Since $\sum_{n=1}^{\infty} \frac{1}{n}$ is a harmonic series, it diverges and so does $\sum_{n=1}^{\infty} a_n$.

(For both, you must state whether the comparable series converges or diverges and why.)

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(1) Use the (direct/basic/ordinary) comparison test to determine whether $\sum_{n=1}^{\infty} \frac{n^2+5^n}{3^n}$ converges or diverges.

Notice $\frac{n^2+5^n}{3^n} > \frac{5^n}{3^n}$ for all $n \in \mathbb{N}$. Since $\sum_{n=1}^{\infty} \frac{5^n}{3^n}$ is a geometric series with $\frac{5}{3} > 1$, it diverges. By the direct comparison test $\sum_{n=1}^{\infty} \frac{n^2+5^n}{3^n}$ diverges as well.

(2) Use the limit comparison test to determine whether $\sum_{n=1}^{\infty} \frac{n^2+3}{8n^3-1}$ converges or diverges.

Let $a_n = \frac{n^2+3}{8n^3-1}$ and $b_n = \frac{1}{n}$. Then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3+3n}{8n^3-1} = \frac{1}{8}.$$

Since $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{8} > 0$, $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or diverge together. Since $\sum_{n=1}^{\infty} \frac{1}{n}$ is a harmonic series, it diverges and so does $\sum_{n=1}^{\infty} a_n$.

(For both, you must state whether the comparable series converges or diverges and why.)

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(1) Use the (direct/basic/ordinary) comparison test to determine whether $\sum_{n=1}^{\infty} \frac{n^3+5^n}{4^n}$ converges or diverges.

Notice $\frac{n^3+5^n}{4^n} > \frac{5^n}{4^n}$ for all $n \in \mathbb{N}$. Since $\sum_{n=1}^{\infty} \frac{5^n}{4^n}$ is a geometric series with $\frac{5}{4} > 1$, it diverges. By the direct Comparison test, $\sum_{n=1}^{\infty} \frac{n^3+5^n}{4^n}$ diverges as well.

(2) Use the limit comparison test to determine whether $\sum_{n=1}^{\infty} \frac{3n^2+n}{20n^4+1}$ converges or diverges.

Let $a_n = \frac{3n^2+n}{20n^4+1}$ and $b_n = \frac{1}{n^2}$. Then

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3n^4+n^3}{20n^4+1} = \frac{3}{20}$.

Since $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{3}{20}$, $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or diverge together. Since $\sum_{n=1}^{\infty} b_n$ is a p-series with $p=2>1$, $\sum_{n=1}^{\infty} b_n$ converges as does $\sum_{n=1}^{\infty} a_n$.

(For both, you must state whether the comparable series converges or diverges and why.)