QUIZ 9 Math 34, Spring 2013

SOLUTIONS

NAME, SECTION

The Maclaurin series for the inverse sine is

$$\arcsin x = \sum_{k=0}^{\infty} \frac{(2k)!}{4^k \cdot (k!)^2 \cdot (2k+1)} x^{2k+1} \quad \text{for} \quad |x| < 1.$$

(1) Using series, find $\lim_{x \to 0} \frac{\arcsin x}{x}$. (Recall that $0! = 1$.)

$$\arcsin x = \frac{1}{1 \cdot 1 \cdot 1} x^1 + \frac{2}{4 \cdot 1 \cdot 3} x^3 + \frac{24}{16 \cdot 4 \cdot 5} x^5 + \left(\text{terms with higher powers of } x\right)$$

$$(\ast) \quad = x + \frac{1}{6} x^3 + \frac{3}{40} x^5 + \left(\text{terms w/ higher powers of } x\right)$$

so $\frac{\arcsin x}{x} = 1 + \frac{1}{6} x^2 + \frac{3}{4} x^4 + \left(\text{terms w/ higher powers of } x\right)$

So $\lim_{x \to 0} \frac{\arcsin x}{x} = 1 + 0 + 0 + 0 = 1$

(2) Using series, find $\lim_{x \to 0} \frac{\arcsin x - x}{x^3}$.

from above $(\ast)$, I see

$$\arcsin x - x = \frac{1}{6} x^3 + \frac{3}{40} x^5 + \left(\text{terms w/ higher powers of } x\right)$$

So $\frac{\arcsin x - x}{x^3} = \frac{1}{6} + \frac{3}{40} x^2 + \left(\text{terms w/ higher powers of } x\right)$

So $\lim_{x \to 0} \frac{\arcsin x - x}{x^3} = \frac{1}{6} + 0 + 0 = \frac{1}{6}$