

Tufts University

Math 12

Department of Mathematics

October 9, 2007

Exam 1

12:00–1:20

No books, notes, or electronic devices such as **calculators, cellphones or personal digital assistants**. You are required to **sign** your exam book. With your signature, you pledge that you have neither given nor received assistance on this exam. Any violations will be reported to the dean, and will result in an F in the course.

An answer alone will not give you full credit; we need to see all your work. Write everything in the bluebook provided, and cross out any work you do not want us to grade. You may keep this exam, but please bring it with you to your next class.

Evaluate the following integrals. Show all your work.

1) (12 pts) $\int \cot^2(t) \sin^2(t) dt$

2) (12 pts) $\int_2^3 2x \ln(x-1) dx$

3) (12 pts) $\int \frac{3x^3 + x^2 + 2x}{x^4 + x^2} dx$

4) (12 pts) $\int \frac{dx}{\sqrt{x^2 - 4x + 20}}$

5) (10 pts) a) Graph the region bounded by $y = \sin(x)$ and $y = 2 - \frac{2}{\pi}x$.

[Hint: First plot for each curve the pairs (x, y) where $x = 0, \pi/2, \pi, 3\pi/2, 2\pi$.]

b) Set-up, but DO NOT EVALUATE, integral(s) for the area of the enclosed region.

6) (17 pts) Determine whether each sequence converges or diverges. If it converges, find the limit justifying your steps. If it diverges, explain why.

a) $a_n = \frac{(-1)^n n^6}{n^7 + 30n^2}$

b) $a_n = \frac{\ln(n)}{6 + \ln(3n)}$

c) $a_n = (-1)^n \cos(1/n)$

Determine if the following integrals are convergent or divergent. Evaluate those that are convergent.

7) (8 pts) $\int_{-\infty}^{\infty} 4x^3 e^{-x^4} dx$

8) (12 pts) $\int_0^{\pi/2} \sec^6 \theta d\theta$ [Hint: First separately evaluate the indefinite integral. If you are unable to do so, at least write the definition of how this specific improper integral is defined.]

9) (5 pts) Determine the FORM of the partial fraction decomposition for

$$\frac{x^3 - 4}{(x^3 - 2x^2 + x)(x^2 + 4)^3}$$

DO NOT EVALUATE THE COEFFICIENTS.