

**Tufts University**  
**Department of Mathematics**  
**Math 12, Test 1**

Monday, October 6, 2008

12:00 p.m.-1:20 p.m.

No calculators, notes, scratch paper or books are allowed. Except where indicated, you must show all your work in your blue book in order to receive full credit. A correct answer with no work might be given zero points. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

For problems 1 to 5, evaluate the definite and indefinite integrals.

1. (12 points):  $\int_{\pi/3}^{\pi/2} \cos^2(6x) dx$

2. (10 points): Do the integral  $\int x^3(16+x^2)^{3/2} dx$  **by trig substitution**. Other methods will not count.

3. (10 points):  $\int e^x \tan^2(e^x) \sec^2(e^x) dx$

4. (10 points):  $\int \frac{11x^2 - 3x - 2}{x(x^2 - 1)} dx$

5. (10 points):  $\int x^4 \ln(x) dx$

6. (10 points): Write out the form of the partial fraction decomposition of the following rational function. **DO NOT COMPUTE THE CONSTANTS.**  $v(x) = \frac{x^4 + 4x^3 + x^2 + 7}{x^2(x+2)^3(x^2+4)(x^2-9)}$

7. (13 points): Determine if the improper integral

$$\int_{-1}^2 \frac{1}{\sqrt[3]{x-1}} dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

8. (9 points): Using the method of cylindrical shells, set up a definite integral for the volume of the solid obtained by rotating the bounded region between the graphs of  $y = 4 - x^2$  and the x-axis about the line  $x = 6$ . **DO NOT EVALUATE THE DEFINITE INTEGRAL.**

**Exam continues on the back of this page.**

9. (6 points): Determine whether each sequence converges or diverges. If it converges, state the limit. You do not need to explain your answers. No partial credit will be given for this problem:

$$(a) a_n = (-2)^n \quad (b) b_n = e^{1/n^2} \quad (c) c_n = \frac{3^n}{(2)^{2n}}$$

10. (10 points): Determine whether each sequence converges or diverges. If it converges, determine the limit. You must provide full reasoning for your answers.

$$(a) a_n = \frac{n^3 + 7}{2n^3 + 6n^2 - n} \quad (b) b_n = 2 - \frac{\cos^2(n)}{n^2}$$