

Tufts University

Math 12

Department of Mathematics

Nov 5, 2007

Exam II

No books, notes **or calculators**. Cross out what you do not want us to grade. You *must* give reasons for full credit. You are required to *sign* your exam book.

1. (12 pts) Determine whether each of the following series converges or diverges. If it converges, find the sum.

$$a) \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right) \quad b) \sum_{n=1}^{\infty} 5 \left(\frac{3^n}{4^{n-1}} \right)$$

In 2 through 4, determine whether the series converges or diverges. Justify your answer, and state any tests, rules and theorems you use.

$$2)(10 \text{ pts}) \sum_{n=1}^{\infty} \frac{10 \sin^2(n)}{n^2}$$

$$3) (10 \text{ pts}) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/4}} \quad 4)(10 \text{ pts}) \sum_{n=1}^{\infty} \frac{n^2 + 12n + 13}{\sqrt{5n^5}}$$

In 5 and 6, determine if the series is absolutely convergent, conditionally convergent, or divergent. Justify your answer, and state any theorems, tests, or rules that you use.

$$5) (10 \text{ pts}) \sum_{n=1}^{\infty} (-1)^n \frac{5n^2 - 1}{2n^3} \quad 6) (10 \text{ pts}) \sum_{n=0}^{\infty} (-1)^n \frac{2(n+1)}{n!}$$

7) (10 pts) Find the radius and interval of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{3^n}$$

Exam continues on the other side...

8) (18 pts) Recall that

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1.$$

Find a power series representation for the following functions and state the radius of convergence.

$$a) f(x) = \frac{x^3}{5+x} \quad b) f(x) = \ln(1+x)$$

9) (10 pts) Find the Taylor series of the function $f(x) = e^{-4x}$ centered at $a = 2$.