No books, notes or calculators are allowed. Cross out what you do not want us to grade. You must show all your work in order to received full credit. You may **not** refer to growth rates when taking limits. Please write neatly and begin each problem on a new page of your blue book. You are required to sign your exam book. With your signature, you pledge that you have neither given nor received assistance on this exam.

1. (10 points) True or False. Please list the letters from (a) to (e) on the first page of your blue book and put your answers to this problem there. Please write the whole word "true" or "false".

   (a) A positive series that converges is absolutely convergent.

   (b) If $\sum_{n=100}^{\infty} a_n$ converges to S, then $\sum_{n=1}^{\infty} a_n$ converges to S.

   (c) $\{1\}_{n=1}^{\infty}$ diverges by the divergence test.

   (d) If $\sum_{n=1}^{\infty} b_n$ diverges and $0 \leq a_n \leq b_n$ for all $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.

   (e) The sequence $\{a_n\}_{n=1}^{\infty}$ diverges if $\lim_{n \to \infty} a_n \neq 0$.

2. (14 points) Determine whether the following sequences converge or diverge. If the sequence converges find its limit. Show all work.

$$(a) \quad a_n = (3+n)^{\frac{1}{n}} \qquad\qquad\qquad (b) \quad a_n = \frac{2^n + 7^n}{4^n + 5^n}$$

3. (10 points) Find the solution $y = y(t)$ of the following initial-value problem. Be sure to solve for y.

$$\frac{dy}{dt} = \frac{e^{2t} + \cos t}{y^2} \quad , \quad y(0) = -1.$$

4. (20 points) Determine whether the following series converge or diverge. If the series converges, find its sum. Justify your answer. State and check hypotheses of any test, rules or theorem you use.

$$(a) \quad \sum_{k=1}^{\infty} (-1)^k \left(\frac{7^k}{4e^k} \right) \qquad\qquad (b) \quad \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k+2}} - \frac{1}{\sqrt{k+1}} \right)$$

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For each of the following problems determine whether the series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorems you use. (10 points each)

5. $\displaystyle\sum_{k=1}^{\infty} \left(\frac{ln(k)}{k^2 + 13} \right)^k$

6. $\displaystyle\sum_{k=1}^{\infty} \frac{20k^3 + 200k^2}{k^5 + 1}$

Determine whether each of the following series diverges, is absolutely convergent, or conditionally convergent. Justify your answer. State and check hypotheses of any test, rules or theorems you use. (10 points each)

7. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{5n!}$

8. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{6n^2 + 10}{n + 10}$

9. (6 points) Estimate the value of the following convergent series with an absolute error less than 10^{-2}. Simplify your answer. (You do not need to show that the series converges.)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{10n^2}$$