Problem Set 7

For a complex number $w = a + bi$, let $h_w : \mathbb{R}^2 \to \mathbb{R}^2$ be the map that takes the input $(x, y)$, forms the complex number $x + yi$, and multiplies by $w$.

For example, if $w = i$, then $h_w((5, 3)) = (-3, 5)$, which you can figure out because

$$i(5 + 3i) = 5i + 3(-1) = -3 + 5i.$$

**Problem A** Show that for any $w$, the map $h_w$ is a linear transformation. (This was defined in class, and you can also find the definition on the bottom of page 146 in the text.)

The fact that it is a linear transformation means that it behaves the same as multiplication by a matrix. For instance when $w = i$ the matrix is

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

In general, for $w = a + bi$, what is the matrix?

**Problem B** For which values of $w$ is $h_w$ an isometry?

Book problems: 5.1.1-3, 5.3.1-4