FIRST MIDTERM

ANSWERS

Math 16A, Section 1, Duchin

PART I: PRE-CALCULUS

Question 1 \((-\frac{3}{5} + \frac{3}{2})^{-2} = \frac{100}{81}\) \(\text{(Simplify)}\)

Question 2 Between which two consecutive integers is each of the following numbers?

\(-2 < -1.83 < -1\)

\(6 < \sqrt{40} < 7\)

\(11 < \sqrt{40 + [5\pi]} < 12\)

Question 3 Let \(f(x) = | -x |.\) \(f(-12) = 12\) \(f(\frac{110}{7}) = \frac{110}{7}\)

True or False?: \(| -x | = x \) \(F\)

Question 4 Simplify \(\sqrt{(x^3 + 1)^2 + 2x^3 + 3}\) as far as possible.

\(\sqrt{x^6 + 4x^3 + 4} \text{ OR } x^3 + 2 \text{ (when } x^2 \leq 0)\)

PART II: FUNCTIONS AND LINES

Question 5 Let \(g(x) = \frac{1}{x^3 + 1}.\)

What is the domain of \(g?\) \(D_g = (-\infty, -1) \cup (-1, \infty)\) \(\text{(same!)}\)

What is the inverse function? \(g^{-1}(x) = \sqrt[3]{\frac{1}{x} - 1}\)

What is its range? \(R_{g^{-1}} = (-\infty, -1) \cup (-1, \infty)\)

Question 6 Find an equation for the line through \(P = (1, 1)\) with slope \(3/4.\)

\(y = \frac{3}{4}x + \frac{1}{4}\)

Find another point \(Q\) on the same line. \(Q = \left(\frac{5}{4}\right)\)

What is the distance between \(P\) and \(Q?\) \(d(P, Q) = \sqrt{5}\)

(or ...)

PART III: LIMITS, CONTINUITY, ASYMPTOTES

Question 7 For each of the following limits, first evaluate the limit.

Then, for each, list all the vertical and horizontal asymptotes which are guaranteed from your answer alone.
(If neither kind, write "none.")

\(\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1} = 3\) \(\text{none}\)

\(\lim_{x \to \infty} \frac{x^2 + 3}{\sqrt{x} + 1} = \infty\) \(\text{none}\)

\(\lim_{x \to 5} \frac{2x}{1 - x} = -8\) \(\text{none}\)

\(\lim_{x \to -\infty} \frac{35x^5 + x^3 + 10000}{x^6 - 1} = 0\) \(\text{horiz. asymp. at } y = 0\)

\(\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = 2x\) \(\text{none}\)

\(\lim_{x \to 10} \frac{1}{(x-10)^2} = \infty\) \(\text{vertical asymp. at } x = 10\)

\(\text{(for example)}\)

Question 8 Find a rational function \(f(x)\) whose graph is the same as the graph of \(g(x) = x^2 + 2\) except that it has a removable discontinuity at \(x = 3.\)

\(f(x) = \frac{(x^2 + 2)(x - 3)}{(x - 3)}\)
**FIRST MIDTERM ANSWERS**

**PART I: PRE-CALCULUS**

**Question 1** \( \left( \frac{3}{4} + \frac{4}{3} \right)^{-2} = \frac{144}{49} \) (simplify)

**Question 2** Between which two consecutive integers is each of the following numbers?

\[-3 < -2.04 < -2 \quad 7 < \sqrt{60} < 8 \quad 22 < \sqrt{60 + [5\pi]} < 23\]

**Question 3** Let \( f(x) = | -x |\). \( f(-12) = 12 \quad f\left(\frac{110}{7}\right) = \frac{110}{7} \) True or False? \( | -x | = x \) \( F \)

**Question 4** Simplify \( \sqrt{(x^3 + 2)^2 + 2x^3 + 5} \) as far as possible.

\( \sqrt{x^6 + 6x^3 + 9} \) \( \text{or} \) \( x^3 + 3 \) \( \text{(when } x^3 + 3 \geq 0) \)

**PART II: FUNCTIONS AND LINES**

**Question 5** Let \( g(x) = \frac{2}{x^3 + 1} \). What is the domain of \( g \)? \( D_g = (-\infty, -1) \cup (-1, \infty) \) \( \text{same!} \)

What is the inverse function? \( g^{-1}(x) = \sqrt[3]{\frac{2}{x} - 1} \)

What is its range? \( R_{g^{-1}} = (\infty, -1) \cup (-1, \infty) \)

**Question 6** Find an equation for the line through \( P = (2, 2) \) with slope \( 3/4 \).

\( y = \frac{3}{4} x + \frac{5}{2} \)

Find another point \( Q \) on the same line. \( Q = (6, 5) \)

What is the distance between \( P \) and \( Q \)? \( d(P, Q) = 5 \) (in this case)

**PART III: LIMITS, CONTINUITY, ASYMPTOTES**

**Question 7** For each of the following limits, first evaluate the limit.

Then, for each, list all the vertical and horizontal asymptotes which are guaranteed from your answer alone. (If neither kind, write "none.")

\[
\begin{align*}
\lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1} &= 4 \quad \text{none} \\
\lim_{x \to \infty} \frac{x^2 + 3}{\sqrt{x} + 1} &= \infty \quad \text{none} \\
\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} &= 2x \quad \text{none} \\
\lim_{x \to 0} \frac{2x}{1 - x} &= -8 \quad \text{none} \\
\lim_{x \to -5} \frac{2x}{1 - x} &= -8 \quad \text{none} \\
\lim_{x \to -\infty} \frac{35x^5 + x^3 + 10000}{x^6 - 1} &= 0 \quad \text{horizontal asymp. at } y = 0 \\
\lim_{x \to 10} \frac{5}{X-10} &= \infty \quad \text{vertical asymp. at } x = 10
\end{align*}
\]

**Question 8** Find a rational function \( f(x) \) whose graph is the same as the graph of \( g(x) = 3x^2 + 1 \) except that it has a removable discontinuity at \( x = 2 \).

\[
f(x) = \frac{(3x^2 + 1)(x - 2)}{(x - 2)}
\]
First Midterm

Answers

Part I: Pre-Calculus

Question 1 \( \left( \frac{2}{5} + \frac{1}{4} \right)^{-2} = \frac{400}{9} \) \(<\text{Simplify}>)

Question 2 Between which two consecutive integers is each of the following numbers?

\[
-5 < -4.88 < -4 \\
8 < \sqrt{77} < 9 \\
23 < \sqrt{77} + \frac{1}{2} < 24
\]

Question 3 Let \( f(x) = | -x | \). \( f(-12) = 12 \) \( f \left( \frac{110}{7} \right) = \frac{110}{7} \) True or False?: \( | -x | = x \) \( \text{F} \)

Question 4 Simplify \( \sqrt{(x^3 + 2)^2 - 2x^3 - 3} \) as far as possible.

\[
\sqrt{x^3 + 2x^2 + 1} \quad \text{(when } x^3 \geq 12) \]

Part II: Functions and Lines

Question 5 Let \( g(x) = \frac{3}{x^3 + 1} \). What is the domain of \( g \)? \( D_g = (-\infty, -1) \cup (-1, \infty) \) \(<\text{Same!}>)

What is the inverse function? \( g^{-1}(x) = \sqrt[3]{\frac{3}{x} - 1} \) What is its range? \( R_{g^{-1}} = (-\infty, -1) \cup (-1, \infty) \)

Question 6 Find an equation for the line through \( P = (2, 1) \) with slope \( \frac{3}{4} \).

\( y = \frac{3}{4} x - \frac{1}{2} \)

Find another point \( Q \) on the same line. \( Q = (6, 4) \) What is the distance between \( P \) and \( Q \)? \( d(P, Q) = 5 \) \(<\text{In this case}>)

Part III: Limits, Continuity, Asymptotes

Question 7 For each of the following limits, first evaluate the limit.

Then, for each, list all the vertical and horizontal asymptotes which are guaranteed from your answer alone. (If neither kind, write “none.”)

\[
\lim_{x \to 1} \frac{x^2 - x - 2}{x - 1} = \text{(DNE)} \quad \text{Vertical asymptote at } x = 1 \\
\lim_{x \to -\infty} \frac{35x^5 + x^3 + 10000}{x^6 - 1} = \text{0} \quad \text{Horizontal asymptote at } y = 0 \\
\lim_{x \to -5^-} \frac{2x}{1 - x} = -8 \quad \text{None} \\
\lim_{x \to \infty} \frac{x^2 + 3}{\sqrt{x} + 1} = \infty \quad \text{None} \\
\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = 2x \quad \text{None} \\
\lim_{x \to 0} \frac{x}{10 - x} = \infty \quad \text{Vertical asymptote at } x = 10 \\
\text{(for example)}
\]

Question 8 Find a rational function \( f(x) \) whose graph is the same as the graph of \( g(x) = x^2 - 2 \) except that it has a removable discontinuity at \( x = 2 \).

\[
f(x) = \frac{(x^2 - 2)(x - 2)}{(x - 2)}
\]