

MATH 180, PROBLEM SET 5

Read Artin's *Algebra*, Chapter 5. We will be covering the material in Sections 1-6, but for now, focus on Sections 1,2,3, and 5. Note that Artin uses the term "operation" where I say action.

- (1) Stabilizers.
 - (a) If G acts on X and $S = \text{Stab}_G(a)$, show that $hSh^{-1} = \text{Stab}_G(ha)$.
 - (b) Let the dihedral group D_5 be generated by a counterclockwise rotation x and a flip y . Supposing that the vertices of a pentagon are labeled counterclockwise from the top 1,2,3,4,5 and that the flip fixes 1, what is the stabilizer of the vertex 3? What is the stabilizer of the top-right edge?
- (2) Let M be the (infinitely generated) group of rigid motions of \mathbb{R}^2 , so that $M = \langle t_a, \rho_\theta, r \rangle$, where t_a is translation by $a = (a_1, a_2)$, ρ_θ is rotation by angle θ about the origin, and r is reflection over the x -axis.
 - (a) For which θ does ρ_θ have finite order?
 - (b) For which α, β is the subgroup $\langle \rho_\alpha, \rho_\beta \rangle \leq M$ a cyclic group? In particular, when $\alpha = a/b$ and $\beta = c/d$ are rationals, does the group have to be cyclic?
 - (c) For which $a, b \in \mathbb{R}^2$ is $\langle t_a, t_b \rangle \cong \mathbb{Z}^2$?
- (3) The upper half-plane.
 - (a) Let $\mathbb{H} = \{z = x + yi : y > 0\} \subset \mathbb{C}$ be the upper half-plane in \mathbb{C} . Show that the $\Gamma = SL_2(\mathbb{R})$ action by fractional linear transformations preserves \mathbb{H} (that is, for all $T \in \Gamma$ and $z \in \mathbb{H}$, the image $T.z$ is also in \mathbb{H}).
 - (b) Let $G \leq \Gamma$ be the cyclic group generated by $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Find the G -orbit of $z = x + yi$. Does any element of G have a fixed point? What is a fundamental domain for the action of G ?
 - (c) Let $G \leq \Gamma$ be the cyclic group generated by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Find the G -orbit of $z = x + yi$. Does any element of G have a fixed point? Find a fundamental domain for the action of G .