COURSE ANNOUNCEMENT: MATH 180
CONTINUED FRACTIONS AND GEOMETRIC CODING

FALL 2006, TU/TH 12:10-1:30PM

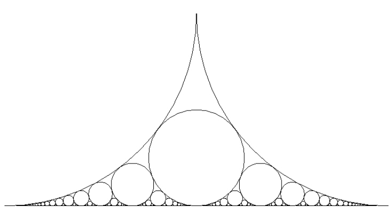
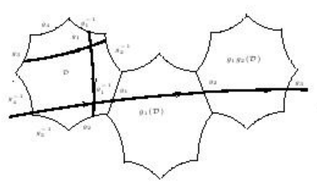
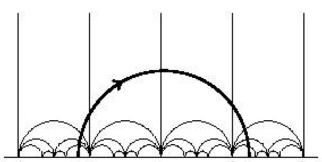
A *continued fraction* is simply a nested fraction – possibly nested infinitely much. In the examples below, α is just a funny way of writing 9/29; on the other hand β gets closer and closer to the irrational value $\sqrt{2}$ as it is expanded farther and farther.

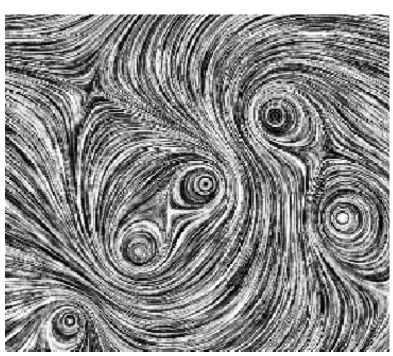
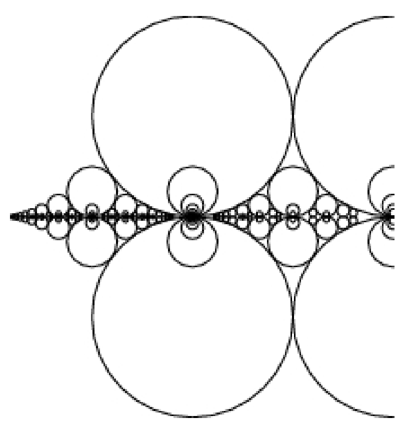
$$\alpha = \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{2}}} \qquad\qquad \beta = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}$$

There's lots of fun number theory to do with continued fractions: for instance, they give a way to find the best *possible* rational approximations to irrational numbers. Also, it is easy to read off from a continued fraction whether the real number that it represents is (a) rational, or (b) a quadratic irrational number (like $\sqrt{2}$, or the golden ratio), but it is unknown whether continued fraction expansions detect other number theoretic properties, like being a cubic irrational number.

A seemingly completely different subject is *dynamical systems*. Suppose you have a way of flowing around in a geometric space and you want to study the properties of the flow. One way to approach this is via *symbolic dynamics*: chop the space up into nice geometric pieces or tiles, and then record how the flow-lines cross through the tiles. This is called a *geometric code*. In this course we will focus on a particular geometric code that has been studied ever since the foundations were laid by Gauss and Hadamard in the 19th century, and we'll find a surprising connection to continued fractions!

Relates to: geometry, topology, combinatorics, dynamics, and number theory.
Prerequisites: calculus and comfort with proofs.

Questions? email the instructor, Moon Duchin (mduchin@math.ucdavis.edu)