

NAME, SECTION

Compute the following integral by the method of partial fractions:

$$\int \frac{3x^2 + 1}{x(x^2 + 1)} dx$$

(a) Set it up for partial fractions by splitting up the expression as a sum of easier-to-integrate terms with some unknowns (A, B, C , etc).

$$\frac{3x^2 + 1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

(b) Solve for the unknowns.

$$\begin{aligned} 3x^2 + 1 &= A(x^2 + 1) + x(Bx + C) \\ &= (A + B)x^2 + Cx + A \end{aligned}$$

So $A=1$, $B=2$ and $C=0$.

(c) Convert the integral and evaluate it.

$$\begin{aligned} \int \frac{3x^2 + 1}{x(x^2 + 1)} dx &= \int \frac{1}{x} + \frac{2x}{x^2 + 1} dx \\ &= \int \frac{1}{x} dx + \int \frac{1}{u} du \quad \left(\begin{array}{l} u = x^2 \\ du = 2x dx \end{array} \right) \\ &= \log|x| + \log|x^2 + 1| + C \end{aligned}$$

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Compute the following integral by the method of partial fractions:

$$\int \frac{2x^2 + 4x + 1}{x(x+1)^2} dx$$

(a) Set it up for partial fractions by splitting up the expression as a sum of easier-to-integrate terms with some unknowns (A, B, C , etc).

$$\frac{2x^2 + 4x + 1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

(b) Solve for the unknowns.

$$\begin{aligned} 2x^2 + 4x + 1 &= A(x+1)^2 + B(x)(x+1) + Cx \\ &= (A+B)x^2 + (2A+B+C)x + A \end{aligned}$$

So $A=1$, $B=1$ and $C=1$.

(c) Convert the integral and evaluate it.

$$\begin{aligned} \int \frac{2x^2 + 4x + 1}{x(x+1)^2} dx &= \int \frac{1}{x} + \frac{1}{x+1} + \frac{1}{(x+1)^2} dx \\ &= \int \frac{1}{x} dx + \int \frac{1}{x+1} dx + \int \frac{1}{u^2} du \quad \left(\begin{array}{l} u = x+1 \\ du = dx \end{array} \right) \\ &= \log|x| + \log|1+x| - \frac{1}{1+x} + C \end{aligned}$$

NAME, SECTION

Compute the following integral by the method of partial fractions:

$$\int \frac{6x^2 + x + 6}{x(x^2 + 1)} dx$$

(a) Set it up for partial fractions by splitting up the expression as a sum of easier-to-integrate terms with some unknowns (A, B, C , etc).

$$\frac{6x^2 + x + 6}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

(b) Solve for the unknowns.

$$\begin{aligned} 6x^2 + x + 6 &= A(x^2 + 1) + (Bx + C)x \\ &= (A + B)x^2 + Cx + A \end{aligned}$$

So $A = 6$, $B = 0$ and $C = 1$.

(c) Convert the integral and evaluate it.

$$\begin{aligned} \int \frac{6x^2 + x + 6}{x(x^2 + 1)} dx &= \int \frac{6}{x} + \frac{1}{x^2 + 1} dx \\ &= 6 \int \frac{1}{x} dx + \int \frac{1}{x^2 + 1} dx \\ &= 6 \log|x| + \tan^{-1}(x) + C \end{aligned}$$