Tufts University

Math 12	**Department of Mathematics**	**December 12, 2008**
	Final Exam	**8:30 – 10:30 a.m.**

No books, notes, **or calculators**. In all problems you must show all your work in order to receive any credit. Write everything in the bluebook provided. Cross out what you do not want us to grade. You are required to **sign** your exam book; with your signature, you pledge that you have neither given nor received assistance on this exam.

Part I. Starting on the first page of your bluebook write the capital letters (A)-(J). For each question below, record only your choice from the indicated list of possible answers. No partial credit is available. Do your work elsewhere.

1. (20 points)

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$ converges. TRUE OR FALSE

(B) If $\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n = 0$. TRUE OR FALSE

(C) Does the sequence $\{\tan^{-1}(n!)\}_{n=1}^{\infty}$ converge or diverge? If it converges, give the limit.

(D) Does the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{2n}}$ converge? If so, give the sum.

(E) Does the series $\sum_{n=1}^{\infty} \frac{\sin^2(n)\sqrt{n}}{n^5+1}$ converge or diverge?

(F) The motion of a particle is given by the parametric equations $x(t) = 3\sin t$ and $y(t) = 2\cos t$. What is the shape of the path the particle follows?

(a) ellipse **(b)** circle **(c)** figure 8 **(d)** spiral

(G) Write down a definite integral that gives the distance traveled by the particle in part (F), as it traces out its path once. **Do not evaluate the integral.**

(H) Solve the equation $\frac{1+2i}{z} + 4 = 1 - i$ for z. Write your answer in the form $z = a + bi$ for real numbers a, b.

(I) Find all complex numbers z for which $z^4 = 16i$. Write your answers in the form $z = re^{i\theta}$ (or $z = r(\cos\theta + i\sin\theta)$) for real numbers r, θ.

(J) On a graph, plot and label the complex solutions to $z^4 = 16i$ found in part (I).

EXAM CONTINUES ON REVERSE.

Part II. For the problems in this part you must show your work in the blue book, giving **full justification**. Cross out any work you do not want graded.

2. (10 points)

(a) Decide whether the improper integral $\int_3^{\infty} \frac{1}{x(\ln x)^{2/3}}\,dx$ converges or diverges.

(b) Use the result of part (a) to decide whether the series $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{2/3}}$ converges or diverges. Make sure to show the series satisfies the hypotheses of the convergence test you are using.

3. (16 points)

(a) Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{x^n}{3^n\sqrt{n}}$.

(b) Find the Taylor series for $f(x) = e^{3x}$ centered at $a = 2$.

4. (21 points) Evaluate the following integrals.

(a) $\int x\ln(2x)\,dx$ **(b)** $\int \frac{x-3}{x^2-3x+2}\,dx$ **(c)** $\int \frac{1}{\sqrt{x^2+4}}\,dx$

5. (7 points) Assume the differential equation $y'' + 4y = 0$ has a series solution $y = \sum_{n=0}^{\infty} c_n x^n$.

You may assume that when you plug $y = \sum_{n=0}^{\infty} c_n x^n$ into $y'' + 4y = 0$, you get:

$$(*) \qquad y'' + 4y = \sum_{n=0}^{\infty} \left[(n+2)(n+1)c_{n+2} + 4c_n\right]x^n = 0.$$

Use this information to solve the initial value problem $y'' + 4y = 0$ with initial conditions $y(0) = 1, y'(0) = 0$ as follows:

(a) Explain why $y(0) = 1$ and $y'(0) = 0$ imply that $c_0 = 1$ and $c_1 = 0$.

(b) Use the facts that $c_0 = 1$ and $c_1 = 0$ and equation $(*)$ (given above) to determine all of the coefficients c_n.

(c) Write the series solution y that solves the initial value problem when $y(0) = 1$ and $y'(0) = 0$.

6. (8 points)

(a) Decide whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{2n+3}$ converges absolutely, converges conditionally, or diverges.

(b) Find an n such that the error between the sum of series in part (a) and the n^{th} partial sum is less than $1/51$, that is error = $|s - s_n| < \frac{1}{51}$. Justify your answer.

EXAM CONTINUES ON NEXT PAGE.

7. (8 points) Assume a radioactive element has a half life of three years. Assume initially there are 12 grams.

(a) How many grams are there after 6 years?

(b) How many grams are there after 10 years?

8. (10 points)

(a) Graph the polar equation $r = 1 - \cos\theta$. Label all points at which your curve intersects the Cartesian axes (i.e. the x- and y-axes); use (r, θ)-coordinates for your labels.

(b) Find the area of the region enclosed by the curve you graphed in part (a).

END OF EXAM.
HAVE A GREAT BREAK!